

On a Theorem of Schmidt

A number of years ago, Wolfgang Schmidt showed that for every distribution of N points in the unit square, there is a convex set in the unit square with discrepancy $\gg N^{1/3}$. In other words, the number of points in the convex set differs from the expectation by $\gg N^{1/3}$.

Schmidt's proof of this beautiful result is very short and elegant. The exponent $1/3$ is also best possible.

In this note, we attempt to explain how Schmidt may have discovered the proof.

LEMMA 1. *Wolfgang Schmidt loves chocolates.*

Proof. Obvious.

LEMMA 2. *Pat Schmidt makes lovely chocolate cakes.*

Proof. Obvious if the reader has been to the Schmidt residence. For others, try to get an invitation to the Schmidt residence.

We now begin the proof.

On Wolfgang's N -th birthday, Pat had made a beautiful round chocolate cake of diameter 1 and placed it on a square plate of area 1. She then decorated this with N chocolates, some of these on top of the cake and others on the plate.

When Wolfgang entered the kitchen while Pat was out, and when he saw the cake, he remembered Lemma 2. It follows from Lemma 1 that Wolfgang decided to cut a small piece. By instinct, he chose to cut a small segment of area $1/2N$, realizing that the remainder would remain convex and that he could repeat this operation $\gg N^{1/3}$ times without destroying the convexity of (what remained of) the cake.

Naturally, it follows from Lemma 1 that those segments that Wolfgang preferred to cut each contained at least one chocolate. After a while, he realized that the remainder of the cake was rather deficient of chocolates. In any case, when Pat returned and discovered that some chocolates were missing, she decided to make another cake, rather similar to the first one. After all, this was Wolfgang's birthday. However, she did put the chocolates closer to the centre of the cake.

Later that day, when Wolfgang saw the second cake, he realized that if he chose again to cut a small segment of area $1/2N$ and repeat this operation a reasonable number of times, these small pieces would now not contain any chocolates, with the result that (what remained of) the cake was still convex but now rather abundant of chocolates.

One way or other, the number of chocolates would differ from the expected number by at least $\gg N^{1/3}$.